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Isotropic points and lines in strain fields

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Abstract—In heterogeneous strain fields, the strain may be isotropic at some points or along some lines called *isotropic points* and *isotropic lines*. These isotropic features are known in various strain environments such as folds, diapirs, nappes or shear zones. Mechanical instabilities involving flow-cells (diapirism and folding) are able to produce simultaneously isotropic points and lines whose dimensions, location and migration are controlled by the flow pattern and history. Many other isotropic points can be understood in terms of superposition of elementary flows. Some examples such as neighbouring diapirs, synkinematic diapirs, shear zone terminations and spreading-sliding nappes are presented. Finally, some three-dimensional aspects and general properties of isotropic points and lines are discussed.

INTRODUCTION

MANY heterogeneous strain fields, in two dimensions, present local points or lines of isotropic strain (i.e. strain ellipses reduced to circles). These strain features here called 'isotropic points' or 'isotropic lines' may be defined also by the strain trajectory patterns around them. *Isotropic points* are distinguishable by their strain trajectories (Fig. 1a). From the common triangular pattern of the trajectories (Fig. 1a) some authors have used the term 'triple points' (Brun *et al.* 1981, Brun & Pons 1981). *Isotropic lines*, however, separate mutually-orthogonal principal trajectories on each side (Fig. 1b). In a plane-strain field, the strain is zero at isotropic points and lines, and they can then be termed neutral points and neutral lines (e.g. Ramsay 1967, p. 416).

As demonstrated by many strain analyses cleavage is quasi-parallel to the $\lambda_1\lambda_2$ plane of the finite strain ellipsoid. Thus mapping of cleavage trajectories is therefore a simple and accurate method for depicting isotropic features in nature.

In this paper, a review is presented of various situations in which isotropic points and lines are known or suspected. An attempt is given to relate strain fields to velocity fields to test similarities and differences between isotropic points and lines in different kinematic environments. General properties of these isotropic features in terms of flow and progressive deformation are discussed.

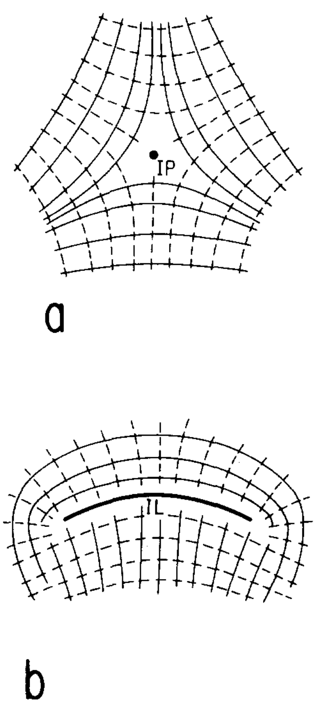

Fig. 1. (a) Idealized isotropic point (*IP*) and (b) isotropic lines (*IL*). Lines, λ_1 trajectories; dotted lines, λ_3 trajectories.

FLOW MODELS

Velocity fields and strain fields

Using the stream-function method (see Jaeger 1969, p. 140, Ramberg 1981 chapters 7 and 9) one can easily compute the velocities and values and orientations of principal strains at any point in a given fluid domain (see Appendix). A number of different models can be obtained by superposition of elementary stream functions. In the rest of the paper, some of these are presented which may be used as qualitative models of natural or experimental situations in which isotropic points and lines are known.

Sources, sinks and doublets

A source is characterized by a radial flow in which the velocity is only dependent on the radius. The stream function is

$$\psi = \frac{Q}{2\pi} \log r \tag{1}$$

where r is the radius and Q the power of the source. For

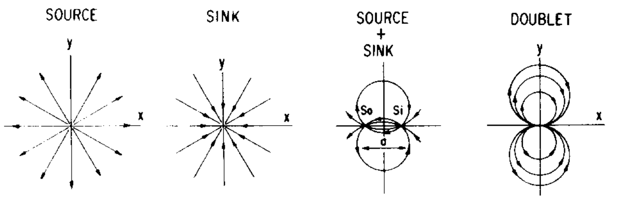

Fig. 2. Flow lines for source, sink, source + sink, and doublet. *So*: source; *Si*: sink.

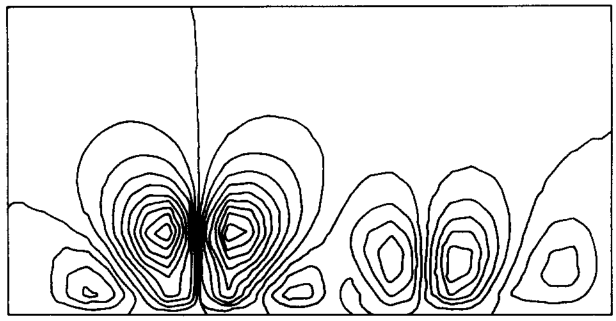

Fig. 4. Flow cells during the early growth of diapirs, after Elder (1977).

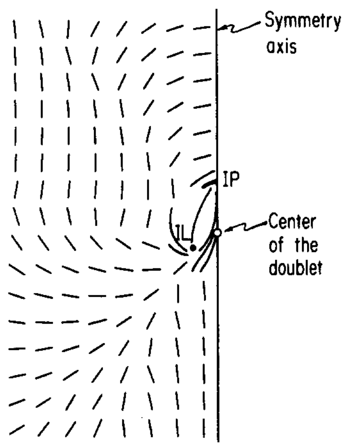

Fig. 3. Strain field due to a doublet. Lengths of bars are proportional to λ_1 values. Near the centre of the doublet principal strain directions have been smoothed by hand. Away from the centre of the doublet, the magnitude of λ_1 varies only slightly. *IP*: Isotropic point; *IL*: Isotropic line.

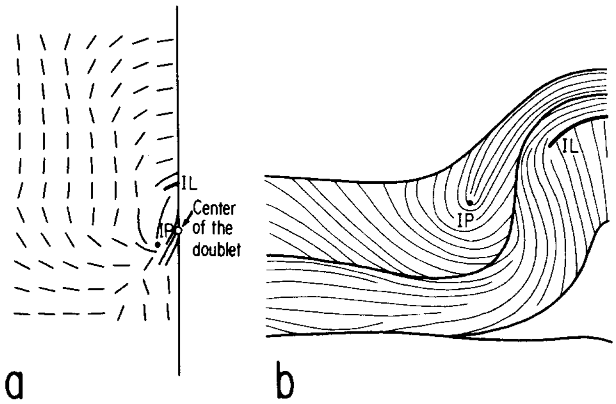

Fig. 5. (a) Strain field resulting from a doublet near a rigid boundary (method of images, see Johnson 1970, pp. 267, 417). (b) λ_1 trajectories in a centrifuged diapir model after Dixon (1975). Viscosity contrast between overburden and source layer: 1/10. *IP*, isotropic point; *IL*, isotropic line.

a source, Q is positive, and for a sink negative (Fig. 2). If we combine a source and a sink separated by a distance a, the flow pattern is characterized by circular stream lines (Fig. 2). If a become very small (i.e. the source is quasi-superposed with the sink), the flow takes the well-known pattern of 'eddies' or flow cells (Fig. 2). For this case, the stream function is:

$$\psi = \frac{\chi}{2\pi} \frac{y}{x^2 + y^2} \tag{2}$$

where x and y are the coordinates of a point, and χ the moment of the doublet.

It is especially interesting to look at the strain pattern produced by a doublet (Fig. 3). We note the existence of an isotropic line and two isotropic points well defined by the principal λ_1 trajectories.

ASSOCIATIONS OF ISOTROPIC POINTS AND LINES RESULTING FROM DOUBLET-TYPE FLOW

Diapirs

Numerical and experimental models of diapirism show that the velocity field during a diapir growth may be simulated by eccentric flow cells (Elder 1977) (Fig. 4). Such a pattern is close to that of a doublet near a rigid boundary (Fig. 5a), and the resulting strain fields (Figs. 5a & b) are comparable, if we exclude some peculiarities due to viscosity contrasts in the experimental models (Dixon 1975).

During the evolution of the diapir, the isotropic line migrates towards the centre of the diapir core and is progressively reduced to a point (Fig. 6). A consequence of this change in the isotropic line configuration is that subvertically stretched areas become progressively horizontally stretched. In rocks, such a kinematic evolution can produce superposed fabric (crenulation of an early plano-linear fabric) giving a material trace of the migration of the isotropic line (Dixon 1975, Schwerdtner 1977, Brun & Choukroune 1981).

Buckle folds

During buckling of a competent layer, the velocity field in the layer can be represented by flow cells. This has been predicted theoretically (Goguel 1948, p. 302) and verified experimentally (Cobbold 1975) (Fig. 7). As buckling progresses the flow cells take on an elongate shape (Fig. 7). Elongate flow cells may be readily obtained if a doublet stream function is superimposed on a 'regional flow' such as pure shear or simple shear. Taking the

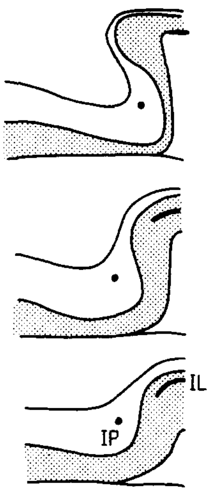

Fig. 6. Evolution of isotropic point (*IP*) and line (*IL*) during a diapir development, after experimental models by Dixon (1975).

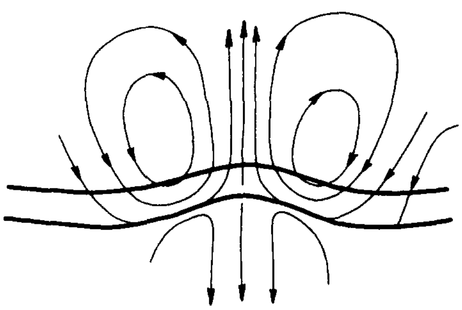

Fig. 7. Flow cells during buckle folding, after an experimental model by Cobbold (1975).

example of a pure-shear doublet superposition, the corresponding stream function is:

$$\psi = \frac{\chi}{2\pi} \frac{y}{x^2 + y^2} + Vxy \tag{3}$$

where the second term is the stream function of the pure shear, V being a velocity constant.

A strain field for particular values of χ and V is given by Fig. 8(a). Only the upper half of the doublet is shown representing an anticline; the neighbouring syncline has a rotational symmetry. We note the existence of one isotropic line and one isotropic point above it. This strain pattern is well known in numerical models (Dieterich 1969), and in experiments (Roberts & Strömgård 1973, Cobbold 1975, Soula 1981) (Fig. 8b), and the corresponding cleavage pattern has been observed in the field (Ramsay 1967, Roberts 1971). These experimental models show that isotropic points and lines migrate during deformation (see for example fig. 15 in Roberts & Strömgård 1973). The same effect can be produced in the stream-function model (equation 3) if we give different lifetimes to the doublet and the pure shear. During the doublet life, the isotropic features migrate slowly outwards from the centre of the doublet. At the death of the doublet, if the pure shear goes further, the isotropic point migrates inwards to meet the isotropic line. If the

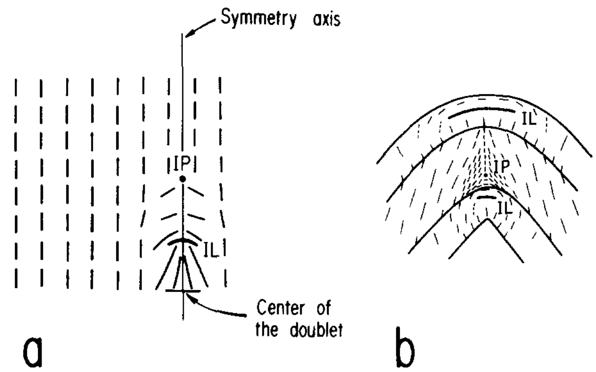

Fig. 8. (a) Strain field resulting from the superposition of a doublet and a pure shear (equation 3). (b) Strain field associated with buckle folds, after an experimental model by Roberts & Strömgård (1973). (*IP*, isotropic point; *IL*, isotropic line; broken lines, λ_1 trajectories.)

pure shear is maintained for a sufficiently long time, the isotropic features disappear. Looking at amplification curves obtained in many experimental models (see Cobbold 1976, fig. 1), we can associate the 'explosive amplification' time during buckling, of Sherwin & Chapple (1968), with the life time of a doublet. The existence of isotropic points and lines in a particular strain pattern may be understood in terms of local and transient perturbations by doublets in a pure shear flow (eventually steady). In theory and experiments, it has been demonstrated that the mechanical amplification depends on the viscosity contrast, which is also a critical factor for the development of the isotropic points (Roberts & Strömgård 1973, Cobbold 1975, Manz & Wickam 1978, Soula 1981). In the present model, the moment of the doublet plays the same role.

As in diapirs, the migration of isotropic features in buckle folds has some interesting structural consequences. The cleavages parallel to the outer arc may be crenulated, indicating inward migration of the isotropic point (see Le Corre 1978, fig. 97).

ISOTROPIC POINTS RESULTING FROM SUPERPOSITION OF SEVERAL ELEMENTARY FLOWS

Synkinematic diapiric plutons

The diapiric emplacement of a pluton in the crust is often characterized by an increase of diameter called the 'ballooning effect' (Ramsay 1975, Holder 1979) or radial distension (Schwerdtner 1972, Morgan 1980). In a horizontal plane, this process may be simulated by a source flow (equation 1). If the pluton emplacement is syntectonic or synkinematic (i.e. synchronous with a regional deformation), a better model may be obtained by superposition of a source and a regional flow. An example of superposition of a source and a simple shear is shown in Fig. 9. The principal stretch trajectories define two isotropic points at the extremities of the source area.

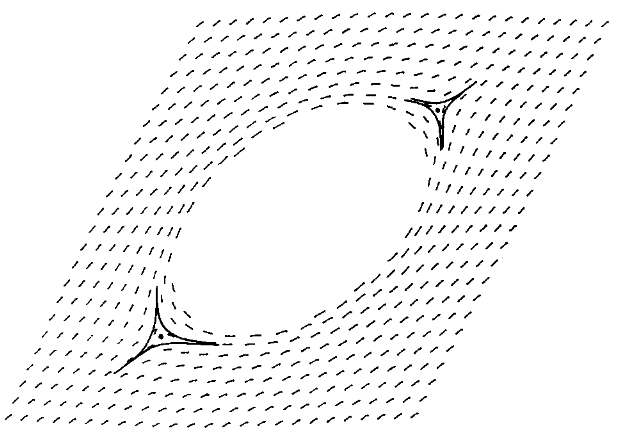

Fig. 9. Strain field resulting from the superposition of a source and a simple shear. λ_1 trajectories (short lines) near isotropic points have been smoothed by hand.

During the source life, the isotropic points migrate outwards from the source centre. If the regional flow is maintained after the source ceases the isotropic points migrate inwards.

Natural and numerical examples of this type have been described by Ledru & Brun (1976) and Brun & Pons (1981) as shown in Fig. 10(b).

Neighbouring diapirs

The juxtaposition of at least three sources gives rise to one isotropic point whose location depends on the relative power of the sources. In the field, such patterns can be observed between neighbouring diapirs (Fig. 10a).

Terminations of shear zones

Ideal shear zones are produced by band-like flow systems (see Cobbold 1977). In most natural cases, shear zones are arranged in complex patterns. One of the main problems associated with various types of shear zones in the field, is the nature of their terminations. Ramsay & Allison (1980) have proposed two theoretical end members one of which, in plane strain, presents isotropic points (Fig. 11). The development of a shear zone corresponds to an elongate vortex-type flow cell (see Coward 1976, fig. 13c and Cobbold 1977, fig. 3a). If the shear zone is isolated, the displacements and resulting strains diffuse in the surrounding of the shear zone and result in curvature at the shear zone ends (see Coward 1976, fig. 13). If the shear zone is straight, the vortex flow may be accommodated at the ends of the shear zone by flow in channels orthogonal or strongly oblique to the shear zone (Fig. 11). The development of an isotropic point in such a system may be understood, as in the previous models, in terms of the superposition of elementary flows (here banded).

Spreading-sliding nappes

Experiments by Brun & Merle (1982) investigated a viscous slab (silicone putty) flowing under its own weight on an inclined plane (Fig. 12). The back part of the slab was only affected by spreading and flowed up-hill (for uphill flow, see Elliott 1976). The rest of the slab was subjected to simultaneous spreading and sliding, and flowed down-hill. At the junction of these two parts, the strain field shows an isotropic point (Fig. 12). The experiments (Brun & Merle 1982) also demonstrate an up-hill migration of the isotropic point as the up-hill flow decreases due to strong thinning of the slab.

This experimental result which has not yet been verified in the field shows, as previously, an isotropic point directly related to the velocity field and whose location, in particular, is controlled by the two different flow systems involved.

THREE-DIMENSIONAL ASPECTS OF ISOTROPIC FEATURES

All the situations considered so far have been two-dimensional. In models, this is done for mathematical or experimental convenience. The terms isotropic lines

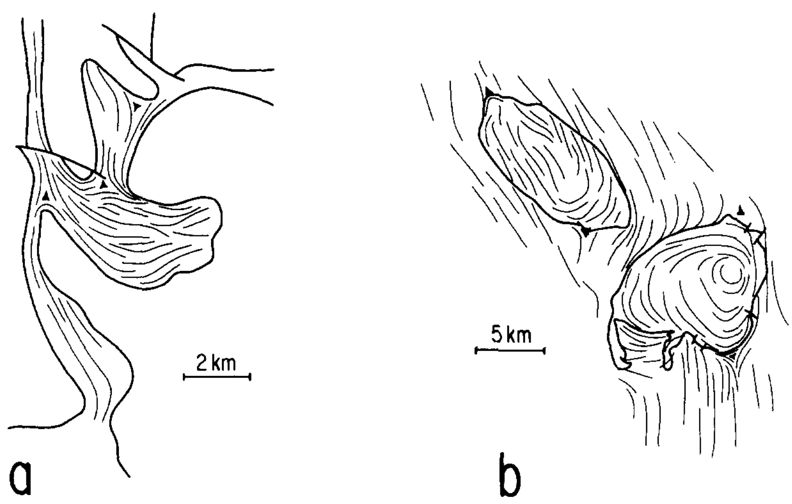

Fig. 10. (a) $\lambda_1\lambda_2$ strike lines between neighbouring mantled gneiss domes (Kuopio, Finland), after Brun *et al.* (1981). (b) $\lambda_1\lambda_2$ strike lines in and around diapiric plutons (Sierra Morena, Spain), after Brun & Pons (1981).

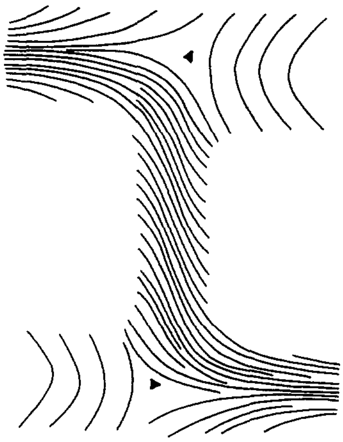

Fig. 11. λ_1 trajectories showing isotropic points at the termination of a shear zone. Plane-strain model after Ramsay & Allison (1979).

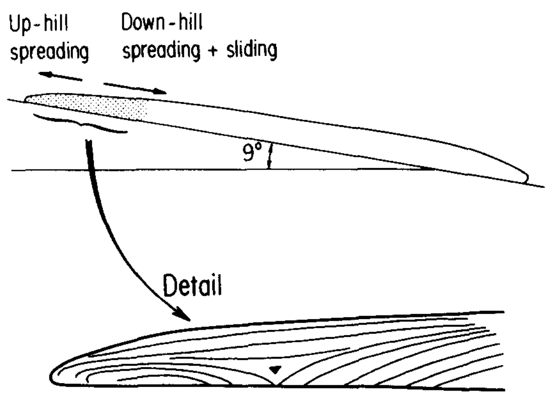

Fig. 12. λ_1 trajectories in a spreading–sliding nappe model flowing on a plane inclined at 9°, after an experimental model by Brun & Merle (1982 and in prep.)

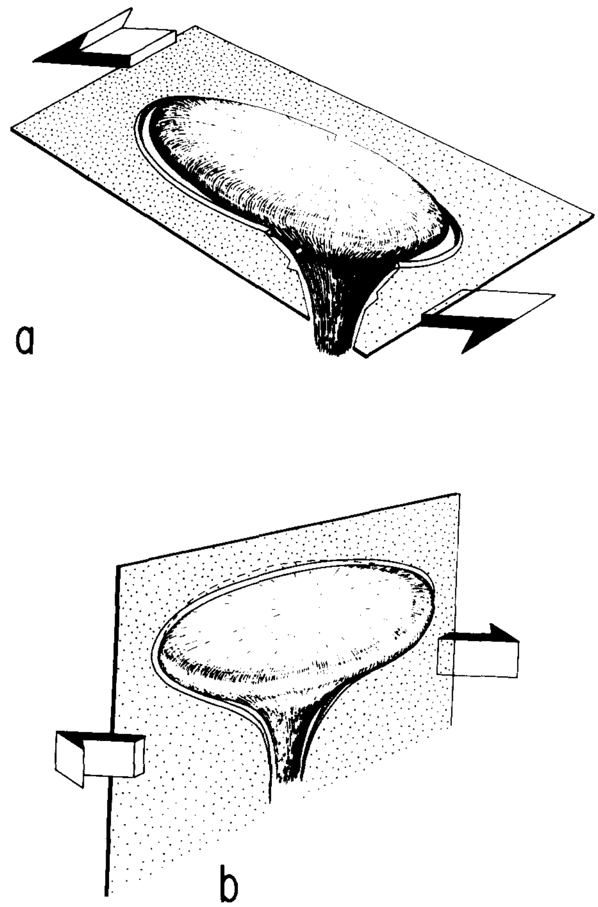

Fig. 13. Three-dimensional sketches of constrictional lines around synkinematic diapirs. (a) Thrusting shear, (b) transcurrent shear. The lines are represented in planes parallel to the mean attitude of $\lambda_1\lambda_2$ surfaces away from the diapirs.

and isotropic points, which describe features in plane view will correspond to surfaces and lines in three dimensions. This is important not only from the terminology point of view. It does have some important consequences as many natural strain fields are not two-dimensional.

On plane sections of three-dimensional strain fields, isotropic points can correspond to sectional ellipses of true flattening or true constrictional strain ellipsoids. In three dimensions, such points are arranged along lines or surfaces that are not inextended. They are lines or surfaces of constriction or flattening. On plane section (e.g. outcrop maps) the principal trajectories around them display the same pattern (Fig. 1) as they do in plane-strain field. Therefore, I suggest that these lines or surfaces may have the same significance in three-dimensional strain fields as true isotropic features do in two-dimensional strain fields. Even if a general discussion of this problem is not possible here, a few examples may be given.

(1) In the case of an interference between the ballooning effect of a diapir and a regional deformation, the isotropic line that develops is stretched along its length as the diapir increases in diameter (Fig. 13). Then the finite strain along the line is of constriction type (Brun 1982).

(2) In one case of natural interfering diapirs, it has been verified by strain measurements that isotropic points (intersections of isotropic lines with the outcrop surface) are located in constriction-type strain areas (cf. Brun *et al.* 1981).

(3) In an axially-symmetric diapir, the isotropic surface that develops is extended in all directions, in its upper part (see Fig. 6), as the diameter of the diapir increases. Thus the finite strain within this surface is of flattening type.

From a theoretical point of view, it is possible that:

(a) an isotropic surface can reduce laterally to give an isotropic line;

(b) there can exist in some environments real isotropic points in three dimensions;

(c) during progressive deformation an isotropic surface can reduce to an isotropic line, and an isotropic line to an isotropic point; and

(d) during progressive deformation an isotropic feature may have a very short life (see for example Hudleston, this issue, fig. 4a).

For these reasons, firstly the term neutral point should be avoided because it implies a plane-strain field. Secondly, even if the detection of isotropic features is easily done by strain-trajectory mapping it should be useful, for three-dimensional interpretation, to have strain measurements as near as possible to isotropic surfaces, points and lines. Thirdly, because isotropic features can migrate and change from a surface to line or a line to a point, or completely disappear during progressive deformation, careful examination of small-scale structures and fabrics in the vicinity of isotropic features is necessary. Such kinematic data should be very important for the understanding of deformation history.

DISCUSSION AND CONCLUSIONS

Mechanical instabilities giving rise to flow cells are able to produce strain fields with isotropic points and lines. From the kinematic point of view, these instabilities may be modelled by a doublet. The development of isotropic points and lines in the strain field can be attributed to the locally divergent and convergent pattern of the flow lines in a doublet. This interpretation is valid for diapirs. In the case of buckle folding, the existence of isotropic lines and points in the finite strain field depends on the relative lifetimes of the doublet flow (perturbation) and the regional flow.

It is important to note that the doublet-type flows is the simplest flow system able to produce an isotropic strain line. In fact, it produces simultaneously two isotropic strain points and one isotropic strain line.

The review presented in this paper shows that isotropic features occur in various tectonic strain fields. An attempt has been made to relate strain patterns to velocity patterns, and in many cases we have seen that isotropic points and lines may be easily understood in terms of simple or composite flows. In this sense, isotropic features are very useful to qualitative interpretation of finite strain fields in terms of progressive deformation. The conclusions are as follows:

(1) Isotropic points and lines can develop simultaneously in a doublet type flow, acting alone (diapirism).

(2) Many superpositions of elementary flows can produce isotropic features:

(a) association of isotropic point and line by doublet perturbation in a regional flow (e.g. buckle folding);

(b) one isotropic point between neighbouring sources (interfering diapirs);

(c) two isotropic points from a source perturbation in a regional flow (synkinematic diapirs) and

(d) junction of two flows (sliding-spreading nappes, shear zone terminations).

(3) In most flow situations, the isotropic points and lines migrate across the medium: they are not material features. If fabrics are produced, it should be possible by careful examination, to obtain some indication of their displacements. Moreover, the spatial stability of isotropic points and lines is subjected to the stability (or life-time) of the involved flows. Thus, data concerning their migration can give direct information on the history of the flow components (mechanical instabilities, spreading in nappes, shear zone propagation, etc.).

(4) Because of the generality of three-dimensional strain in nature, many so-called isotropic features could be in fact true constriction or the flattening features. Strain measurements should indicate if this is the case.

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REFERENCES

Brun, J. P. 1981. Instabilités gravitaires et déformation de la croûte continentale. Application au développement des dômes et des plutons. *Thèse d'état*. Rennes.

Brun, J. P. & Choukroune, P. 1981. Déformation progresssive et structures crustales. *Rev. Géogr. phys. Géol. dyn.* **23**, 177–193.

Brun, J. P., Gapais, D. & Le Theoff, B. 1981. The mantled gneiss domes of Kuopio (Finland): interfering diapirs. *Tectonophysics* **74**, 383–304.

Brun, J. P. & Pons, J. 1981. Strain patterns of pluton emplacement in a crust undergoing non coaxial deformation. *J. Struct. Geol.* **3**, 219–229.

Brun, J. P. & Merle, O. 1982. La Nappe du Parpaillon (Flysch à helminthoïdes de l'Embrunais Ubaye): II Mécanisme de mise en place. *9ème Réun. Ann. Sci. Terre, Paris* 98.

Cobbold, P. R. 1975. Fold propagation in single embedded layers. *Tectonophysics* **27**, 333–351.

Cobbold, P. R. 1976. Fold shapes as functions of progressive strain. *Phil. Trans. R. Soc.* **A283**, 129–138.

Cobbold, P. R. 1977. Description and origin of banded deformation structures—II. Rheology and the growth of banded perturbations. *Can. J. Earth Sci.* **14**, 2510–2523.

Dieterich, J. H. 1970. Computer experiments on mechanics of finite amplitude folds. *Can. J. Earth Sci.* **7**, 467–476.

Dixon, J. M. 1974. A new method of determining finite strain in models of geological structures. *Tectonophysics* **24**, 99–114.

Dixon, J. M. 1975. Finite strain and progressive deformation in models of diapiric structures. *Tectonophysics* **28**, 89–124.

Elder, J. W. 1977. Thermal convection. *J. geol. Soc. Lond.* **133**, 293–309.

Elliott, D. 1976. The motion of thrust sheets. *J. geophys. Res.* **81**, 949–963.

Goguel, J. 1948. Introduction à l'étude mécanique des déformations de l'écorce terrestre. *Mém. Carte géol. Fr.* 2nd ed., Paris.

Holder, M. T. 1979. An emplacement mechanism for post-tectonic granites and its implications for their geochemical features. In: *Origin of Granite Batholiths, Geochemical Evidence* (edited by Atherton, M. P. & Tarney, J.). Shiva Publishing Limited, Orpington, U.K., 116–128.

Hudleston, P. J. 1983. Strain patterns in an ice cap and implications for strain variations in shear zones. *J. Struct. Geol.* **5**, 455–463.

Jaeger, J. C. 1969. *Elasticity, Fracture and Flow*. Methuen, London.

Johnson, A. M. 1970. *Physical Process in Geology*. Freeman, Copper & Cie, San Francisco.

Le Corre, C. 1978. Approche quantitative des processus synchisteux. L'exemple du segment hercynien de Bretagne Centrale. *Thèse d'état*. Rennes.

Ledru, P. & Brun, J. P. 1977. Utilisation des fronts et des trajectoires de schistosité dans l'étude des relations entre tectonique et intrusion granitique: exemple du granite de Flamanville (Manche). *C. r. hebd. Séanc. Acad. Sci., Paris* **258**, 1199–1202.

Manz, R. & Wickam, J. 1978. Experimental analysis of folding in simple shear. *Tectonophysics* **44**, 79–90.

Morgan, J. 1980. Deformation due to the distension of cylindrical igneous contacts: a kinematic model. In: *Analytical Studies in Structural Geology* (edited by Schwerdtner, W. M., Hudleston, P. J. & Dixon, J. M.). *Tectonophysics* **66**, 167–178.

Ramberg, H. 1981. *Gravity, Deformation and the Earth's Crust*, 2nd edition. Academic Press, London.

Ramsay, J. G. 1967. *Folding and Fracturing of Rocks*. McGraw-Hill, New York.

Ramsay, J. G. 1980. Emplacement mechanics of the Chindamora batholith, Zimbabwe. *Tectonic Studies Group Meeting, Leeds*, Abstract.

Ramsay, J. G. & Allison, I. 1979. Structural analysis of shear zones in an Alpinised Hercynian granite. *Schweiz. miner. petrogr. Mitt.* **59**, 251–279.
Roberts, D. 1971. Abnormal cleavage patterns in fold hinge zones from Varanger Peninsula, Northern Norway. *Am. J. Sci.* **269**, 170–180.
Roberts, D. & Strömgård, K. E. 1973. A comparison of natural and experimental strain patterns around fold hinges. *Tectonophysics* **14**, 102–105.
Schwerdtner, W. M. 1972. Behaviour of an Archean granitic batholith: Discussion. *Can. J. Earth Sci.* **9**, 1230–1234.
Schwerdtner, W. M. 1977. Geometric significance of regional strain analysis. *Tectonophysics* **39**, 515–531.
Sherwin, J. A. & Chapple, W. M. 1968. Wavelengths of single-layer folds: a comparison between theory and observation. *Am. J. Sci.* **266**, 164–179.
Soula, J. C. 1981. Plissement experimental de monolits et de multilits à contraste de compétence faible ou moyen; comparaison avec des structures naturelles. *Rev. Geogr. phys. Géol. dyn.* **23**, 235–256.

APPENDIX

Computation of flow models

For plane non-inertial strain of an incompressible Newtonian fluid if a given stream function ψ, satisfies the biharmornic equation:

$$\frac{\partial^4\psi}{\partial x^4} + 2\,\frac{\partial^4\psi}{\partial x^2 y^2} + \frac{\partial^4\psi}{\partial y^4} = 0, \qquad (4)$$

the velocities are given by:

$$u = -\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = \frac{\partial\psi}{\partial x}, \qquad (5)$$

and one can easily compute the corresponding velocity field. For details concerning the derivation of equations (4) and (5) and other applications see Jaeger (1969, p. 140) and Ramberg (1981, chapters 7–9).

Taking an initially-square grid, finite displacements of the nodes have been computed by iterations. Then, orientations and values of principal strains have been computed for each element of the deformed grid.

All the computed flow models presented in the paper are steady flows. The number of iterations used to obtain a finite-strain pattern depends on the magnitude of the increments which depend themselves on the nature and place of the constants used in the stream functions (cf. Q, χ, V in equations 1–3).

Complex flow patterns have been obtained by summation of elementary stream functions (superposition law).

Near the singularities of velocity fields (e.g. the centre of a doublet), the elements of the grid lost their parallelogram shape. Therefore, in these areas principal strains have been determined graphically and smoothed by hand (see Fig. 3).